TEST CODE **01234020**

FORM TP 2015089

MAY/JUNE 2015

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has EIGHT questions and Section II has THREE questions.
3. Answer ALL questions in Section I, and any TWO questions from Section II.
4. Write your answers in the booklet provided.
5. Do NOT write in the margins.
6. All working MUST be shown clearly.
7. **A list of formulae is provided on page 2 of this booklet.**
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
9. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

0123402003

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Arc length — $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of a sector — $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

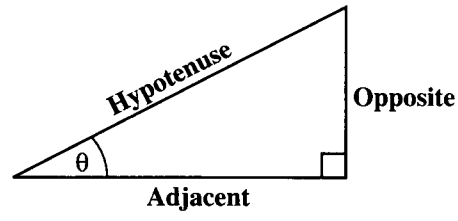
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

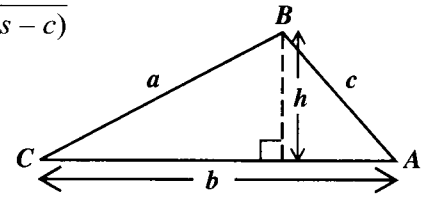


Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$



Sine rule — $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$

GO ON TO THE NEXT PAGE

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the EXACT value of:

(i) $2\frac{2}{5} - 1\frac{1}{3} + 3\frac{1}{2}$

(1 mark)

(ii) $(4.14 \div 5.75) + (1.62)^2$

(2 marks)

(iii) $2 \times 3.142 \times 1.25$

(1 mark)

(iv) $\sqrt{2.89} \times \tan 45°$

(2 marks)

(b) The table below shows a shopping bill prepared for Mrs Rowe. The prices of some items are missing.

Shopping Bill		
Item	**Unit Cost Price**	**Total Cost Price**
3 kg sugar	**X**	\$10.80
4 kg rice	**Y**	**Z**
2 kg flour	\$1.60	\$3.20

(i) Calculate the value of **X**, the cost of 1 kg of sugar.

(1 mark)

(ii) If the cost price of 1 kg of rice is 80 cents MORE than for 1 kg of flour, calculate the values of **Y** and **Z**.

(2 marks)

(iii) A tax of 10% of the total cost price of the three items is added to Mrs Rowe's bill. What is Mrs Rowe's TOTAL bill including the tax?

(3 marks)

Total 12 marks

GO ON TO THE NEXT PAGE

2. (a) Given that $a = 4$, $b = 2$ and $c = -1$, find the value of:

(i) $a - b + c$

(1 mark)

(ii) $2a^b$

(1 mark)

(b) A bottle contains **500 ml** of orange juice. Write an expression for EACH of the following.

The amount of juice left in the bottle after pouring out

(i) p ml

(1 mark)

(ii) q glasses each containing r ml.

(1 mark)

(c) Write as a single fraction, as simply as possible

$$\frac{2k}{3} + \frac{2-k}{5}.$$

(2 marks)

(d) Four mangoes and two pears cost \$24.00, while two mangoes and three pears cost \$16.00.

(i) Write a pair of simultaneous equations in x and y to represent the information given above.

(2 marks)

(ii) State clearly what x and y represent.

x represents .....

.....

y represents .....

.....

(1 mark)

(e) Factorize completely:

(i) $a^3 - 12a$

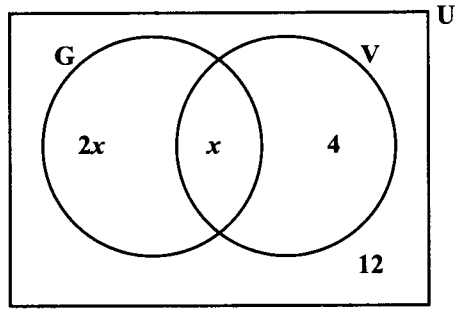
(1 mark)

(ii) $2x^2 - 5x + 3$

(2 marks)

Total 12 marks

3. (a) The Venn diagram below shows the number of students who play the guitar (G) or the violin (V) in a class of 40 students.



(i) How many students play neither the guitar nor the violin?

(1 mark)

(ii) Write an expression, in terms of x, which represents the TOTAL number of students in the class.

(1 mark)

(iii) Write an equation which may be used to determine the total number of students in the class.

(1 mark)

(iv) How many students play the guitar?

(2 marks)

GO ON TO THE NEXT PAGE

(b) (i)Using **a ruler, a pencil and a pair of compasses**, construct triangle ABC with AB = 9 cm, angle ABC = 90° and BC = 6 cm.

(4 marks)

(ii)Measure and state the size of angle BAC.

(1 mark)

(iii)On the diagram, show the point D such that ABCD is a parallelogram.

(2 marks)

Total 12 marks

NOTHING HAS BEEN OMITTED.

GO ON TO THE NEXT PAGE

4. A graph sheet is provided for this question.

The table below is designed to show values of x and y for the function $y = x^2 - 2x - 3$ for integer values of x from -2 to 4.

x	-2	-1	0	1	2	3	4
y	5		-3	-4	-3		5

(a) Complete the table for the function $y = x^2 - 2x - 3$. **(2 marks)**

(b) **On the graph on page 13**, plot the graph of $y = x^2 - 2x - 3$ using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis. **(4 marks)**

(c) **On the graph on page 13**, draw a smooth curve passing through the points on your graph. **(1 mark)**

(d) Complete the following sentences using information from your graph.

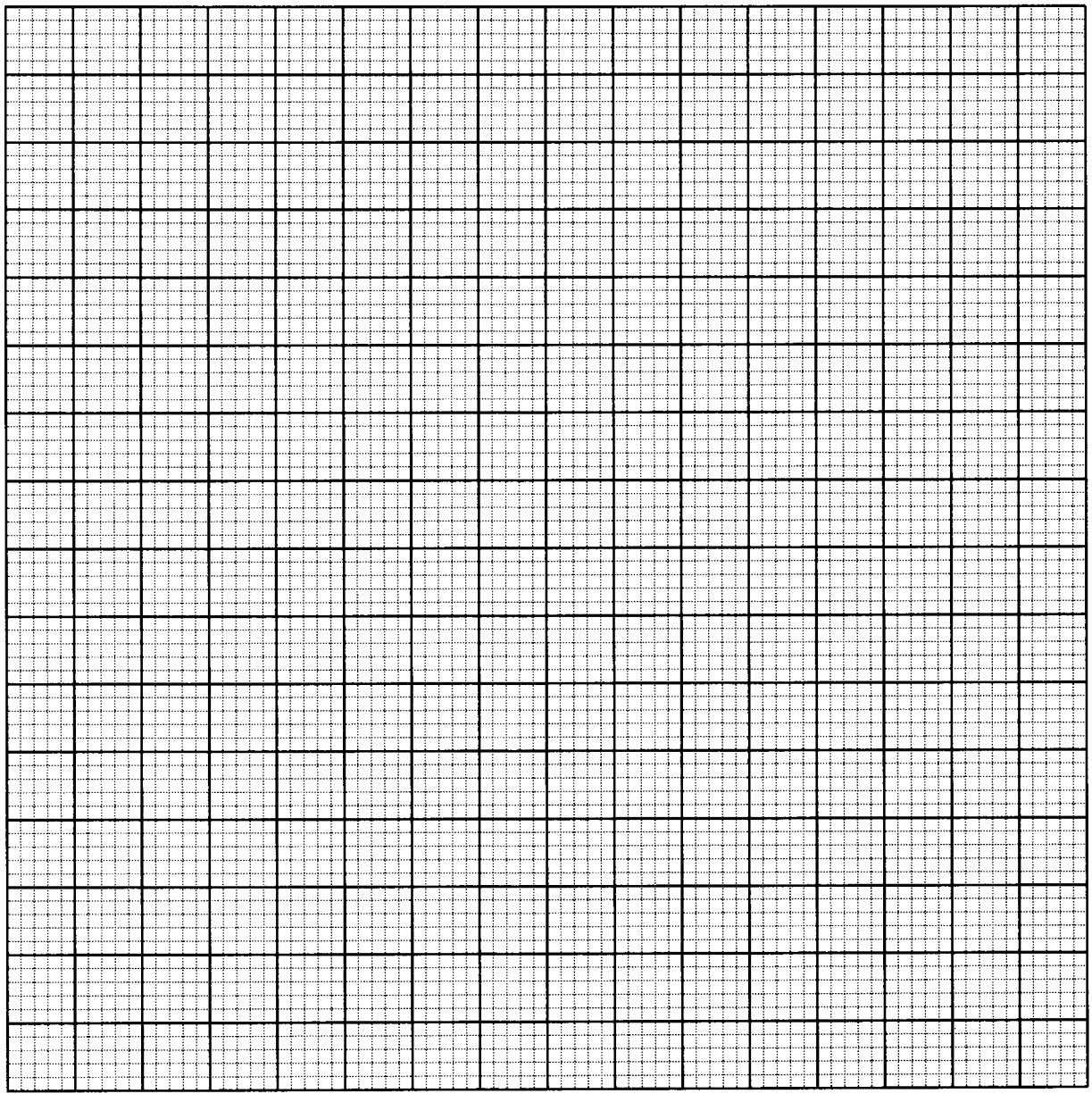
(i) The values of x for which $x^2 - 2x - 3 = 0$ are ..... and ..... . **(1 mark)**

(ii) The **minimum** value of $x^2 - 2x - 3$ is ..... . **(1 mark)**

(iii) The equation of the line of symmetry of the graph of

$y = x^2 - 2x - 3$ is ..... . **(1 mark)**

Total 10 marks



5. (a) A car is travelling at a constant speed of 54 km/h.

(i) Calculate the distance it travels in $2\frac{1}{4}$ hours.

(1 mark)

(ii) Calculate the time, in seconds, it takes to travel 315 metres, given that

$$1 \text{ km/h} = \frac{5}{18} \text{ m/s}.$$

(2 marks)

(b) Write the following scales in the form $1 : x$.

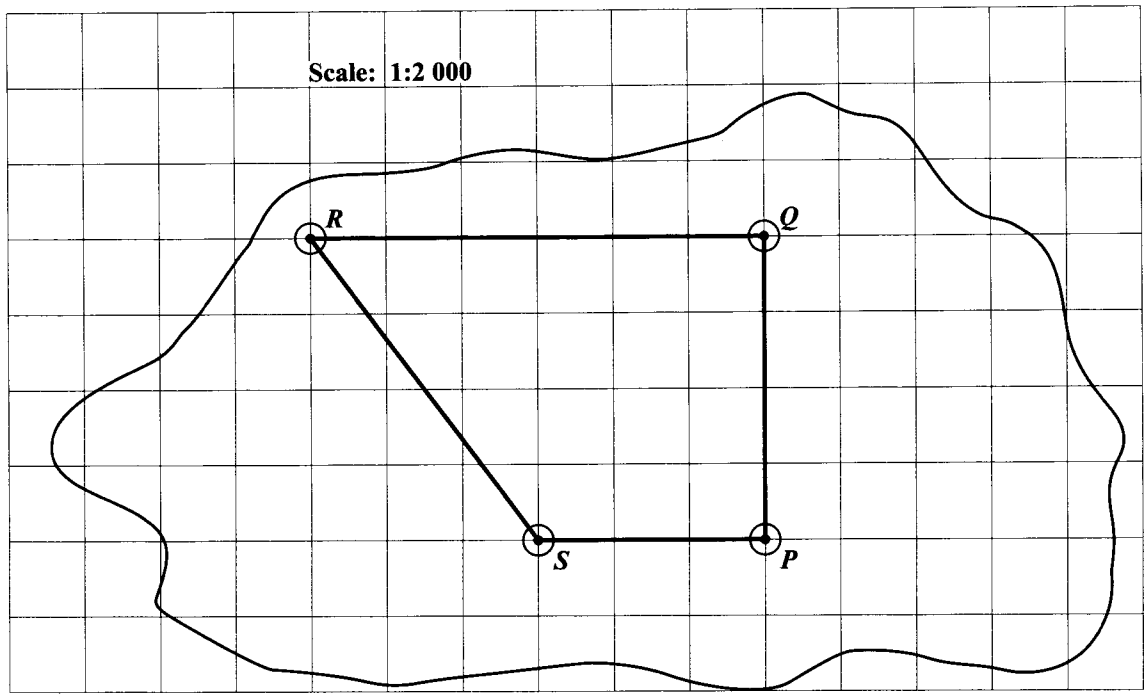
(i) 1 millimetre = 1 metre

(1 mark)

(ii) 2 cm = 6 m

(1 mark)

(c) The map shown below is drawn on a grid of 1 cm squares. P, Q, R and S are four tracking stations. **The scale of the map is 1:2 000.**



(i) Determine, in centimetres, the distance from Q to R on the map.

QR = ... cm **(1 mark)**

(ii) Determine, by counting, the area in square centimetres of the plane $PQRS$ on the map.

(2 marks)

(iii) Calculate the ACTUAL distance, in kilometres, between Q and R.

(2 marks)

GO ON TO THE NEXT PAGE

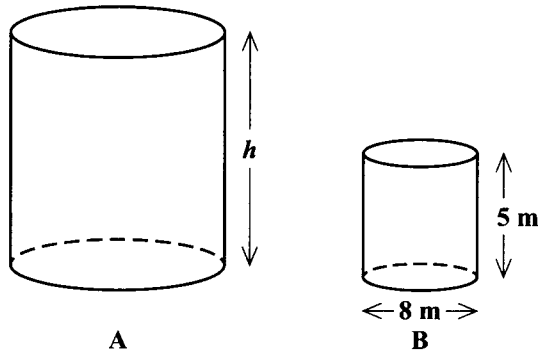
(iv) Calculate the ACTUAL area, in square metres, of the plane PQRS.

(2 marks)

Total 12 marks

NOTHING HAS BEEN OMITTED.

6. (a) The diagram below, **not drawn to scale**, shows two cylindrical water tanks, A and B. Tank B has base diameter 8 m and height 5 m. Both tanks are filled with water.



Take $\pi = 3.14$.

(i) Calculate the volume of water in Tank B.

(2 marks)

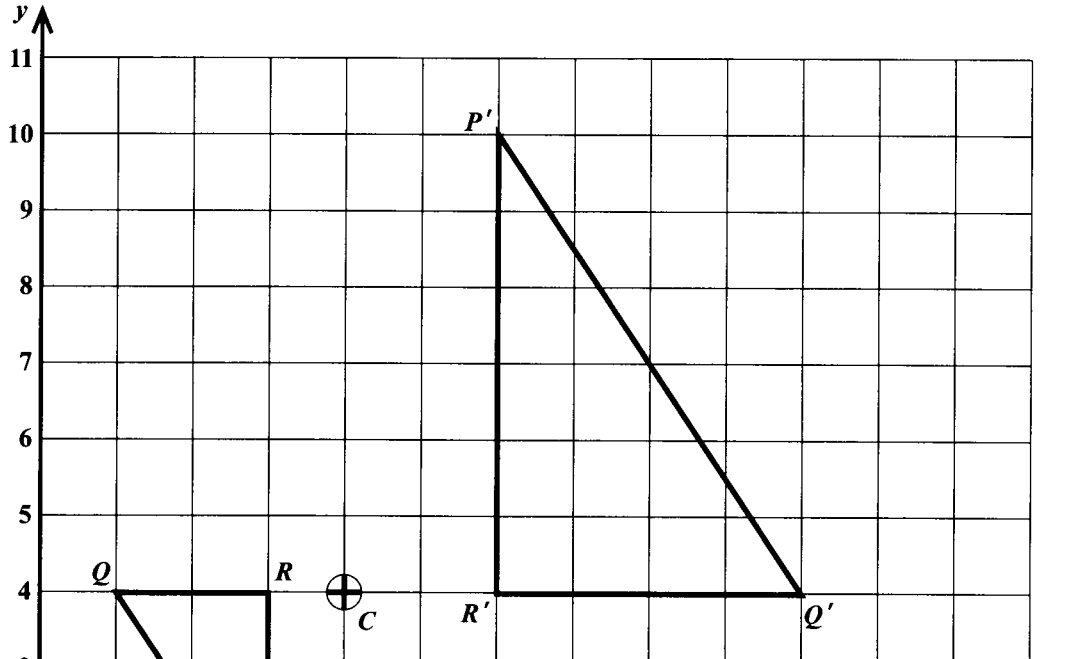
(ii) If the area of the base of A is 314 m^2, calculate the length of the radius of Tank A.

(1 mark)

(iii) Tank A holds 8 times as much water as Tank B. Calculate the height, h, of Tank A.

(2 marks)

(b) The diagram below shows triangle PQR and its image, triangle $P'Q'R'$, after an enlargement centred at the point C on the diagram.



Use the information from the diagram to complete the statements below.

(i) The size of the scale factor is ..... . **(1 mark)**

(ii) The scale factor is negative because .....

.....

.....

(1 mark)

(iii) The length of PQ is $\sqrt{13}$ units, therefore,

the length of $P'Q'$ is ..... units.

(1 mark)

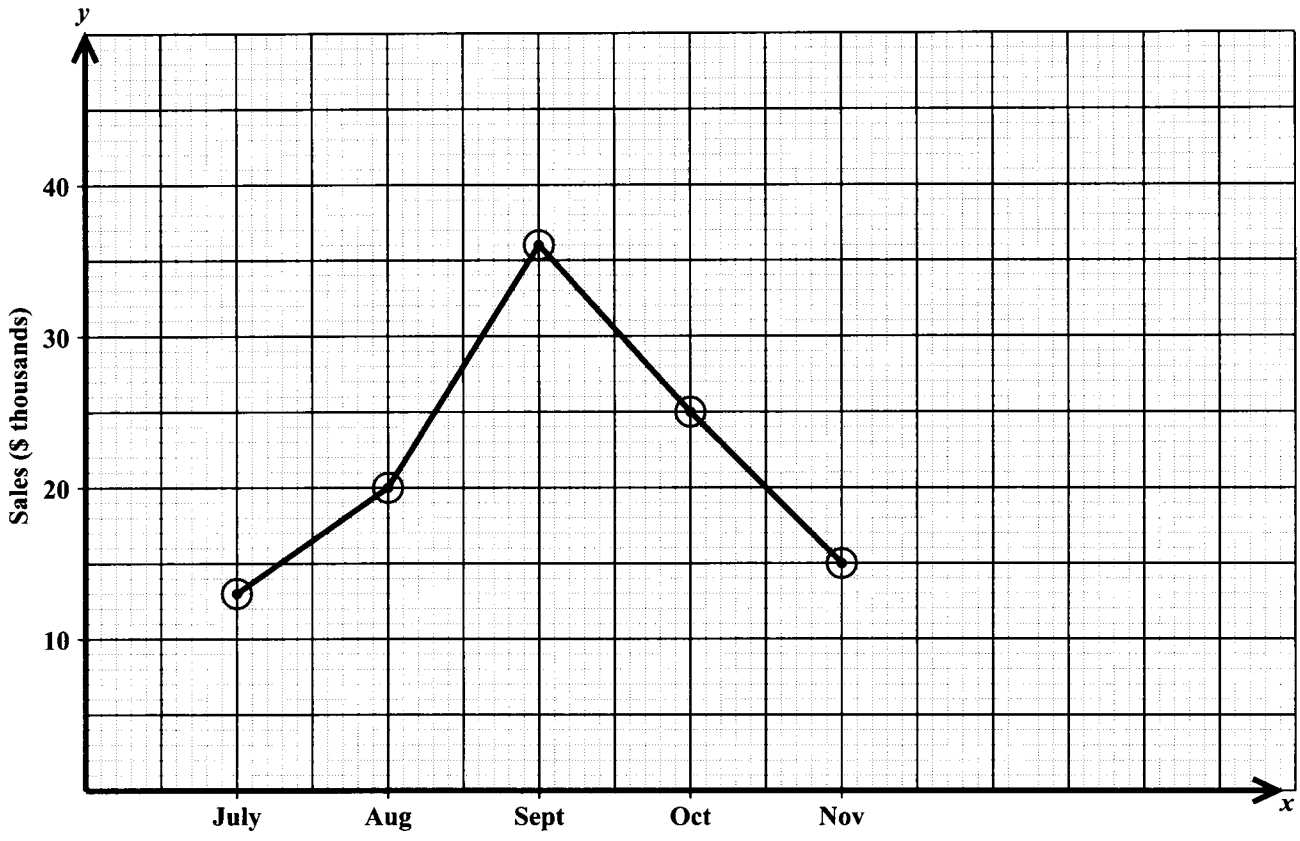
(iv) The area of triangle PQR is ..... square units.

(1 mark)

(v) The area of $P'Q'R'$ is ..... times the area of triangle PQR which is ..... square units. **(2 marks)**

Total 11 marks

7. The line graph below shows the monthly sales, in thousands of dollars, at a car dealership for the period July to November 2014.



(a) Complete the table below to show the sales for EACH month.

Month	July	August	September	October	November
Sales in $ Thousands	13		36		

(2 marks)

(b) (i) Between which TWO consecutive months was there the GREATEST increase in sales?

..... and .....

(1 mark)

(ii) Between which TWO consecutive months was there the SMALLEST increase in sales?

..... and .....

(1 mark)

(iii) What feature of the line graph enables you to infer that the increase in sales between two consecutive months was the greatest or the smallest?

.....

.....

.....

.....

(2 marks)

(c) Calculate the mean monthly sales for the period July to November 2014.

(2 marks)

(d) The TOTAL sales for the period July to December was $130 000.

(i) Calculate the sales, in dollars, for the month of December.

(1 mark)

(ii) Complete the line graph to show the sales for December. **(2 marks)**

Total 11 marks

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8. A sequence of figures is made up of equilateral triangles, called unit triangles with unit sides. The first three figures in the sequence are shown below.

Figure 1

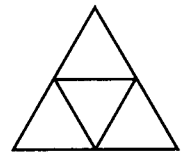


Figure 2

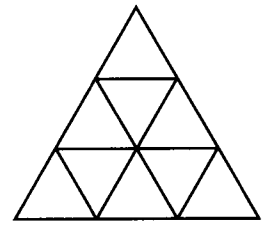


Figure 3

(a) Draw Figure 4 of the sequence.

(2 marks)

(b) Study the patterns of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures. Some rows have not been included in the table.

Complete the rows numbered (i), (ii), (iii) and (iv).

Figure	Number of Unit Triangles	Number of Unit Sides
1	1	$\frac{(3 \times 1)(1 + 1)}{2} = 3$
2	4	$\frac{(3 \times 2)(2 + 1)}{2} = 9$
3	9	$\frac{(3 \times 3)(3 + 1)}{2} = 18$
4		
	144	
25		
n		

(i) **(2 marks)**

(ii) **(2 marks)**

(iii) **(2 marks)**

(iv) **(2 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) **A graph sheet is provided for this question.**

A teacher marks an examination out of a maximum of 120 marks. The marks are then converted to percentages.

(i) Calculate the percentage for a student who scores

- 60 marks

- 120 marks.

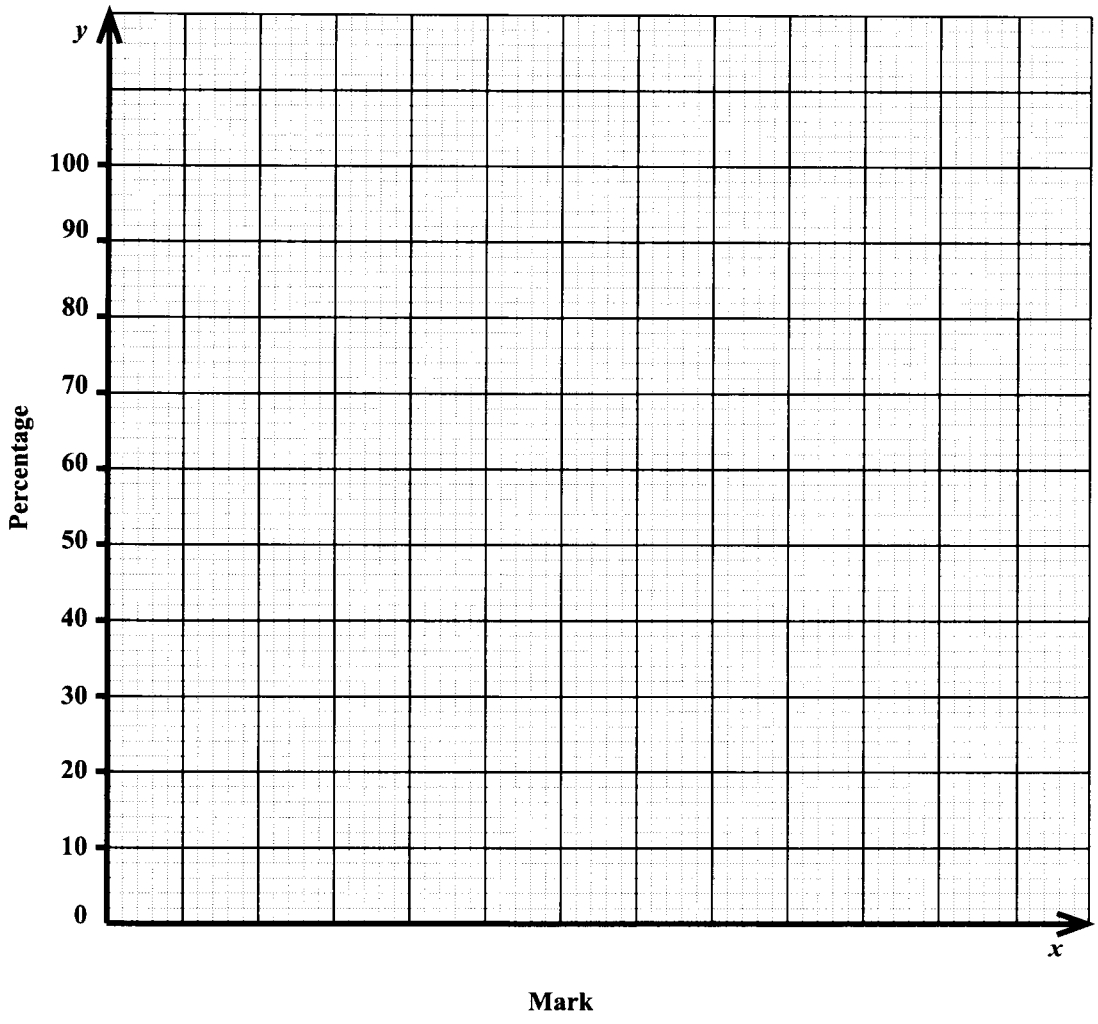
(2 marks)

(ii) **On the graph sheet on page 27**, plot a graph to show the information in (i).
(2 marks)

(iii) A candidate is awarded 95 marks on the examination. Use the graph drawn at (ii) to determine the candidate's percentage.

Draw lines on your graph to show how the percentage was obtained.

Percentage: .....
(1 mark)



(iv) A candidate is awarded a Grade A if her percentage is 85% or more. Use the graph drawn at (ii) to determine the minimum mark the candidate needs to be awarded a Grade A.

Draw lines on your graph to show how the percentage was obtained.

Minimum mark: .....

(2 marks)

(b) The functions $f(x)$ and $g(x)$ are defined as

$$f(x) = 3x + 2 \qquad g(x) = \frac{x^2 - 1}{3}.$$

(i) Evaluate $g(5)$.

(2 marks)

(ii) Write an expression in terms of x for $f^{-1}(x)$.

(2 marks)

(iii) Write an expression for $gf(x)$, in the form $(x + a)(x + b)$, where a and $b \in R$.

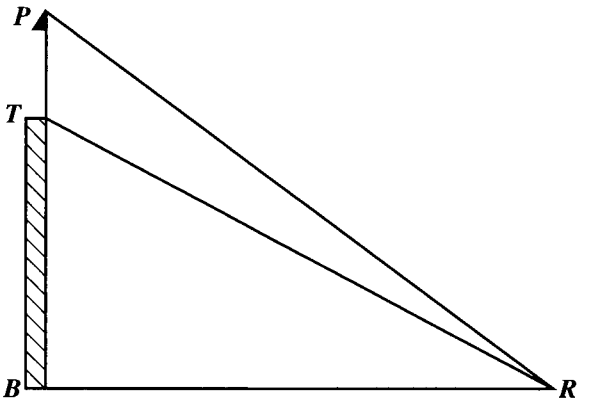
(4 marks)

Total 15 marks

GO ON TO THE NEXT PAGE

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a vertical tower, BT, with a flagpole, TP, mounted on it. A point R is on the same horizontal ground as B, such that $RB = 60$ m, and the angles of elevation of T and P from R are 35° and 42°, respectively.



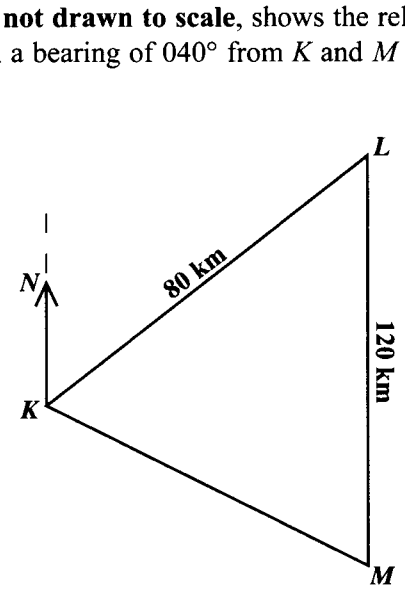
(i) Label the diagram to show

- the distance 60 m
- the angles of 35° and 42°
- any right angle(s). **(3 marks)**

(ii) Calculate the length of the flagpole, giving your answer to the nearest metre.

(4 marks)

(b) The diagram below, **not drawn to scale**, shows the relative positions of three fishing boats, K, L and M. L is on a bearing of 040° from K and M is due south of L. $LM = 120$ km and $KL = 80$ km.



(i) On the diagram show the bearing of 040°. **(1 mark)**

(ii) Calculate the measure of $\angle KLM$.

(1 mark)

(iii) Calculate the length, to the nearest kilometre, of KM.

(3 marks)

GO ON TO THE NEXT PAGE

(iv) Calculate the measure of $\angle LKM$ to the nearest degree.

(2 marks)

(v) Calculate the bearing of M from K.

(1 mark)

Total 15 marks

VECTORS AND MATRICES

11. (a) (i) Calculate the matrix product **AB** where $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(2 marks)

(ii) Show that the matrix product of **A** and **B** is NOT commutative, that is, $\mathbf{AB} \neq \mathbf{BA}$.

(2 marks)

(iii) Find $\mathbf{A}^{-1}$, the inverse of **A**.

(2 marks)

(iv) Given that $\mathbf{M} = \begin{pmatrix} 2x & 2 \\ 9 & 3 \end{pmatrix}$, calculate the value(s) of x for which $|\mathbf{M}| = 0$.

(2 marks)

(b) The position vectors of the points R, S and T, relative to an origin, O, are $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ respectively.

(i) Calculate the value of $|\overrightarrow{OR}|$.

(1 mark)

(ii) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, the vectors $\overrightarrow{RS}$ and $\overrightarrow{ST}$.

$\overrightarrow{RS} =$

$\overrightarrow{ST} =$

(2 marks)

(iii) Using the results of combining the vectors in (b) (ii) **on page 33**, justify that RS is parallel to ST and that RST is a straight line.

(2 marks)

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

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Question No. ☐

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.